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Final Report on Subcontract B605152: Multigrid Methods for Systems of PDEs

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Progress has been made in several of the proposed areas of study, as summarized below.

Aggressive coarsening, polynomial smoothing and Fourier Analysis

The project team has continued with work on developing aggressive coarsening techniques for AMG methods. Of particular interest is the idea to use aggressive coarsening with polynomial smoothing. Using local Fourier analysis the optimal values for the parameters involved in defining the polynomial smoothers are determined automatically in a way to achieve fast convergence of cycles with aggressive coarsening. Numerical tests have the sharpness of the theoretical results. The methods are highly parallelizable and efficient multigrid algorithms on structured and semi-structured grids in two and three spatial dimensions.

Auxiliary Space Preconditioners for the Overlap Operator

We additionally continued with our research on the design of a defect-correction type method based on the idea to use the Wilson system to precondition the overlap operator. We have conducted extensive tests for the 4d overlap operator. In the free case, the method exhibits optimal performance, however, in the gauged case the method is less efficient. In this more general setting it's performance is also nearly optimal for increasing problem sizes or varying masses and gives more than an order of magnitude speed up over the best algorithms currently available for the overlap operator. In addition, we have shown that the non-normality of the Wilson system can be measured using Wilson's action for the pure gauge since this corresponds to the Frobenius norm of D^TD – DD^T, with D the discretized Wilson operator. In this regard, smearing techniques which aim to minimize the gauge action also improve on the normality and hence performance of the preconditioner. Applying the defect correction scheme together with smearing techniques for the Wilson system leads to a highly efficient preconditioner in both 2d and 4d.

Bootstrap Algebraic Multigrid.

We continued to develop adaptive Bootstrap AMG (BAMG) solvers for the non-Hermitain Wilson Dirac system. The approaches employ an optional red-black preconditioner on the finest level with Bootstrap AMG solver for the resulting coarse system. We explored the method when the red-black preconditioner is not used because this is more suitable for the Hypre implementation we are now working on, since then standard full-coarsening is simple to implement in Hypre.

The BAMG setup algorithm used to compute interpolation is based on the notion of least squares fitting of test vectors, i.e, a set of vectors computed adaptively using in addition a multilevel eigensolver. This is an alternative, and potentially far less expensive approach to the adaptive aggregation-based setup algorithm we originally developed for the Wilson system. The adaptive aggregation-based setup algorithm generates the prototypes used in defining Multigrid interpolation sequentially, which requires computing the MG hierarchy at each adaptive step. In contrast, our BAMG setup computes interpolation in a single (or few) adaptive step(s) using as a main tool a multilevel eigensolver. Additionally, we have derived both Petrov-Galerkin and Galerkin versions and shown their equivalence. These new BAMG approaches have been shown to yield a solver whose performance is better than that of the aggregation based scheme with significantly less worked required in the setup.

More recently, we developed

<u>References</u>

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